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# DEVELOPMENT OF SMALL PERTURBATIONS IN A SLIGHTLY NONPARALLEL SUPERSONIC FLOW


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Presently the linear theory of stability of plane-parallel flows of a compressible fluid has been rather well developed [1]. Flows encountered in practice are nonuniform in space as a rule. Often one can assume this nonuniformity to be weak (for example, the flow in a boundary layer). In recent years several alternatives have been developed for the construction of a solution when the average parameters of the flow vary weakly in some directions. The first theoretical results for a boundary layer of incompressible fluid which take account of nonparallelness of the flow were obtained in [2, 3]. An experimental check of the conclusions obtained in those papers was performed in [4]. The development of perturbations in a supersonic boundary layer with nonparallelness taken into account was discussed theoretically in [5-7]. Two-dimensional perturbations were treated in [5], and perturbations of a more general kind were treated in [6, 7]. These papers give significantly different results for the very same conditions. Thus, a strong effect of nonparallelness on the stability characteristics is obtained in [6], but a weak effect is obtained in [5, 7]. There are no experimental papers formulated


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with the purpose of checking the conclusions of the theory. One of the fundamental achievements of the theory which takes into account nonparallelness of the flow in a boundary layer consists of the fact that it has proven possible to calculate the dependence of the degree of growth of the perturbations $\alpha_i$ on the coordinate $\eta$ normal to a streamlined surface. Measurements are made in [8] of the degree of growth of perturbations for three values of $\eta$ with the Mach number M = 2.2. These measurements show that nonparallelness of the flow affects the nature of the development of the perturbations, but it is impossible to draw any kind of conclusions even about a qualitative agreement of theory and experiment. The dependence of the growth coefficients on the transverse coordinate seems more convenient for comparison of theoretical and experimental results. The problem is also posed in this paper of performing such measurements and comparing them with calculations made on the basis of the approach developed in [7].

1. The experiments were conducted in the T-325 wind tunnel of the Institute of Theoretical and Applied Mechanics of the Siberian Branch of the Academy of Sciences of the USSR [9] with a cross section of the working section of 200 × 200 mm in the case of the unit Reynolds number $Re_1 = 5 \cdot 10^6$ m$^{-1}$ at a Mach number M = 2.

The measurements were performed in the boundary layer formed on a steel plate (the same one as in [10]), which was exposed at zero angle of attack in the central plane of the working section of the wind tunnel. The leading edge of the plate is tapered so that the blunt radius is less than 0.02 mm.

The development of the perturbations was recorded with the help of a TPT-2 constant-current thermoanemometer [11]. Resistance unifilar detectors made out of gold-plated tungsten with a wire diameter of 6 $\mu$m were used.

The experimental procedure is similar to that of [8, 10]. With the maximum value of the longitudinal coordinate x measured from the leading edge of the plate downstream the thermoanemometer detector was exposed at some distance from the plate surface, and the dimensionless Blasius coordinate $\eta = (y/x)\mathrm{Re}$ was calculated, where $\mathrm{Re} = (\mathrm{Re}_1 x)^{1/2}$. Then the detector was moved upward through the flow so that the average voltage in the diagonal of the thermoanemometer bridge E was kept constant (by virtue of movement in the y-coordinate); the detector was moved along a line of equal velocities and temperatures and was in a state of constant sensitivity.

In connection with the movement of the detector, the thermoanemometer signal was recorded by a video tape recorder, and then an amplitude–frequency analysis of the recording was made, which permitted obtaining the growth curves of the perturbation of a specified frequency f, i.e., the dependence of the mean square voltage oscillations on the wire of the thermoanemometer detector on the longitudinal coordinate $\langle e_f \rangle = \varphi(x)$. For a control, the thermoanemometer signal was fed in parallel with the tape recorder to a spectrum analyzer, and the growth curve of the perturbations for one of the frequencies was constructed directly in the experiment.

The growth coefficients of the perturbations $\alpha_i$ were determined for x = 100 mm from the relationship

$$\alpha_i = -0.5 d(\ln\langle e_f \rangle)/d\mathrm{Re}.$$

For this the growth curves were measured in the interval from $x_1 = 90$ mm to $x_2 = 110$ mm with a step of 2.5 mm. A second-order approximating curve was constructed from the points obtained using the method of least squares, and the value of the derivative at the point x = 100 mm was calculated.

2. The calculations were performed for a boundary layer on a flat heat-insulated plate at M = 2.0 in accordance with the experiments which were conducted. The temperature law of the viscosity was adopted from Satterland, the adiabatic index $\gamma = 1.4$, and the Prandtl number $\sigma = 0.72$. Since the flow in the boundary layer is self-similar, the dimensionless coordinates $\eta = y/\delta(x)$, $\xi = \int_0^x dx/\delta(x)$, $\zeta = U_\infty z/\nu_\infty$, $\tau = U_\infty^2 t/\nu_\infty$, were introduced, where $\delta(x) = \sqrt{x\nu_\infty/U_\infty}$. A solution for the perturbations was sought in the form $q = q_0(\xi, \eta)\exp[\Theta(\xi, \zeta, \tau)]$. A specially developed procedure [7] permitted determining the logarithmic derivative of q with respect to $\xi$ and thereby the growth rate of the perturbations

$$\alpha_i = \mathrm{Real}\ \{\Theta_\xi + (1/q_0)dq_0/d\xi\}.$$

By virtue of the fact that $q_0$ is a function of $\eta$, the growth rate also depends on the transverse coordinate. In addition it is different for different perturbation parameters (for example, see [5]).

Let us direct our attention to the following fact. A solution $q_0$ is being sought as similar to the eigenfunction of the theory of the plane-parallel approximation which depends on frequency and the wavelengths in the $\xi$ and $\zeta$ directions, i.e., on $\Theta_\tau$, $\Theta_\xi$, and Re $\Theta_\zeta$. In experiments with natural perturbations, the frequency $\Theta_\tau$ is completely determined. In order to determine the four additional parameters (the imaginary and real parts of $\Theta_\xi$ and $\Theta_\zeta$), one makes use of the complex dispersion relationship. Therefore two more conditions are necessary for complete determinacy of the problem. In the case of natural perturbations there is no growth in the z direction. Therefore, it was assumed in the calculations that Real(Re $\Theta_\zeta$) = 0. The relationship

$$\mathrm{Im}(\mathrm{Re}\Theta_\zeta) = \mathrm{Im}(\Theta_\xi)\mathrm{tg}\chi$$

was adopted as the last condition, where $\chi$ is the angle of the propagation direction of the perturbations. Since $\chi$ was not measured in the experiment, the calculations were performed for several values of $\chi$.

3. A comparison of the neutral curves (the condition $\alpha_i = 0$ is satisfied for them) obtained in this paper for $y/\delta = 0.5$ (Fig. 1, points 1) with the results of [12] (continuous curves obtained by averaging a large number of experimental points) is presented in Fig. 1. Here $\delta$ is the thickness of the boundary layer. The letters denote the following: a) neutral curve produced by the interaction of sound emitted by the turbulent boundary layer of the working section of the wind tunnel with the boundary layer of the model (see [1] for more detail); b) lower branch of the curve of neutral stability; and c) upper branch of the curve of neutral stability. Good agreement of the data is obtained.

Data obtained for $y/\delta = 0.7$ (points 2) are also presented in Fig. 1. They are in good agreement with the results of measurement of the neutral curve (dashed curves) presented in [8] (M = 2.2, $y/\delta = 0.7$-$0.8$).

For $y/\delta = 0.5$ the perturbations in the boundary layer reach maximum values and the range of unstable frequencies is a maximum. The instability region on the outer edge of the boundary layer bounded by the neutral curve becomes appreciably narrower. This decrease in the instability region for $y/\delta = 0.7$ is one of the indications of nonparallelness of the flow in the boundary layer.

It is caused by the fact that as the Reynolds number increases the distribution of the perturbations representing the natural oscillations is deformed in the boundary layer (perturbations of the Toulmin–Schlichting wave type) so that the maximum of the perturbations is shifted towards the surface of the streamlined body. This was noted already in [8]. The fact that the positions of the first maximum (curve a) for $y/\delta = 0.5$ and 0.7 agree is more interesting, and one can conclude that nonparallelness of the flow does not affect the position of the neutral point for sonic perturbations.

The theory developed in [7] takes into account the effect of nonparallelness of the flow on the development of natural oscillations of the boundary layer. The interaction of sound with the boundary layer weakens as the Reynolds number increases, but the natural oscillations grow and make the main contribution to the thermoanemometer signal at large values of Re. Therefore the measurement of the growth coefficients of the perturbations was performed at a sufficiently large Reynolds number Re = 648 in order that one could neglect sound. The experimental values of $\alpha_i$ (points 4) are given in Fig. 2 for different $\eta$ with $F = 0.38 \cdot 10^{-4}$ and Re = 648. The dependence of the growth coefficients of the perturbations of the mass flow rate $\alpha_i$ on the transverse Blasius coordinate was calculated for propagation angles of the perturbations of $\chi$ = 60, 42, and 30°, $F = 0.38 \cdot 10^{-4}$, and Re = 640 (numbers 1-3 on Fig. 2, respectively). The measurement results are in qualitative agreement with theory and quantitatively closest of all to the calculations for $\chi = 42°$.

One should note that the propagation angle of the perturbations was not controlled in the experiments with natural perturbations. Measurements conducted earlier [13] have shown that in free flow through the working section of the T-325 wind tunnel perturbations are caused by narrowly directed radiation from the turbulent boundary layer at the walls; for M = 2.0 the radiation angle is equal to 42°.

It has usually been assumed when comparing theoretical calculations with the experimental data for natural perturbations (where the propagation angles are known) that perturbations growing at the maximum rate are present in the laminar boundary layer. Evidently this is inadmissible for wind tunnels. If the radiation is narrowly directed and the radiation angle is less than the angle corresponding to perturbations growing at the maximum rate, then waves emanating from the side (with respect to the model) walls and the corners can cause oblique comparatively slowly growing perturbations of sufficiently great intensity, which may be the cause of the transition. Under our conditions perturbations with a propagation angle of 42° may be such perturbations. At present there is practically no experimental information on the propagation of oblique perturbations in supersonic boundary layers, and the formulation of the appropriate experiments seems to be the most important problem.

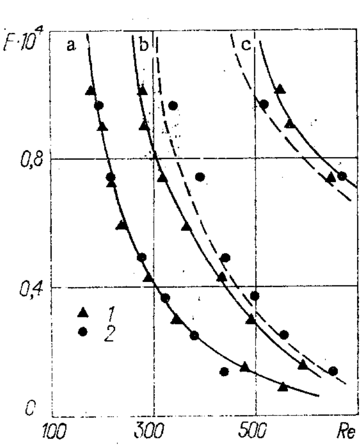


Fig. 1

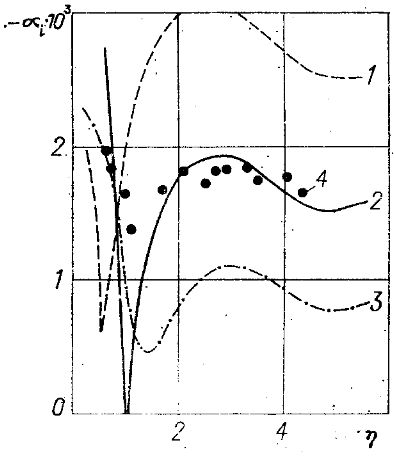


Fig. 2

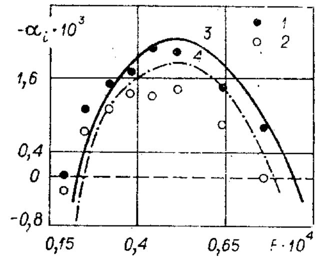


Fig. 3

The dependences of the growth coefficients of perturbations on the frequency parameter are given in Fig. 3 for two values of $y/\delta$. The experimental points are obtained for Re = 648, and the theoretical curves are calculated for Re = 640 and $\chi = 42°$; 1 and 3 correspond to $y/\delta = 0.55$, and 2 and 4 correspond to $y/\delta = 0.7$. The larger the growth coefficients of the perturbations are in the layer with maximum oscillations of the mass rate ($y/\delta = 0.55$), the narrower is the range of unstable frequencies.

The experimental measurements of the growth coefficients of the perturbations as a function of the frequency parameter and the transverse coordinate qualitatively confirm the computational results, but experimental information on the propagation of oblique perturbations in supersonic boundary layers is necessary for a more correct comparison.

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